Resistivity Questions for AS Physics

Q1. (a) A pencil "lead" is made from non-metallic material which has a resistivity, at room temperature, of $4.0 \times 10^{-3}\ \Omega$m. A piece of this material has a length of 20mm and a diameter of 1.40 mm.

Show that the resistance of this specimen, to two significant figures, is 52 Ω.

$$\rho = \frac{RA}{L}$$

$A = \pi r^2 = \pi (1.40 \times 10^{-3}/2)^2 = 1.54 \times 10^{-6}\ m^2$ **(1 mark)** $L = 20 \times 10^{-3}$ m

$R = \rho L/A =$ **(1 mark)** $4.0 \times 10^{-3} \times 20 \times 10^{-3} / 1.54 \times 10^{-6} = 52\Omega$ **QED**

(2)

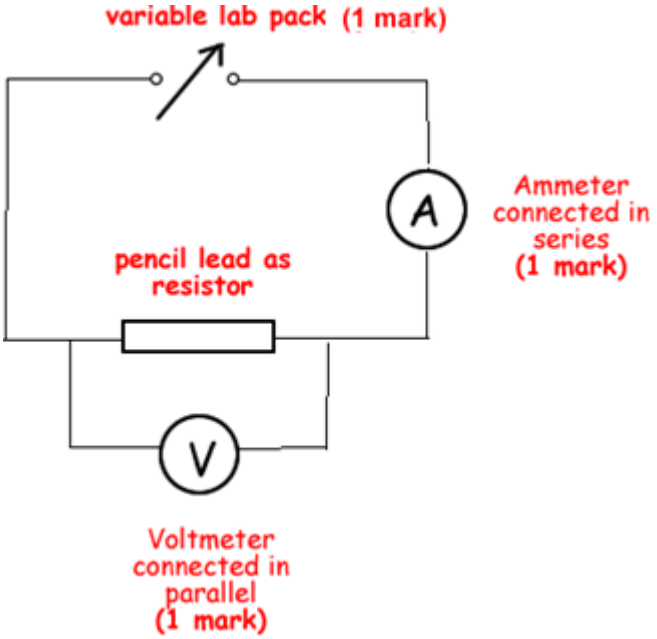
(b) Given a specimen of the pencil "lead" described in part (a) with similar dimensions, describe an experiment you could carry out in the school or college laboratory to verify that the resistivity of the material is equal to the value quoted in part (a).

Your description should include

- a labelled circuit diagram,
- details of the measurements you would make,
- an account of how you would use your measurements to determine the result.

The piece of lead you have been given is only 20 mm long (2.0 cm is not very big!). It would not be possible to vary the length as we did with a metre of wire… the most you could manage is two length readings from it at the most!

You would therefore not be able to plot a graph of length against resistance – but you could find resistance by varying current – but not too high because of heating effect.



Measure the diameter of the pencil lead with a micrometer screw gauge. **(1 mark)** Repeat this three times at different orientations and take an average. **(1 mark)** Use this value to work out the cross sectional area of the pencil lead using the eqution $A = \pi(d/2)^2$ **(1 mark)**

Measure the length of the pencil lead with vernier calipers. **(1 mark)** Use them to measure in between the crocodile clip connections. Ensure that the clips are perpendicular to the shaft of the lead when you take the reading. **(1 mark)**

Wire up the circuit as shown in the diagram. Steadily increase the voltage from the power supply from zero until the current is 0.10A. Note down the readings from the voltmeter and ammeter. **(1 mark)**. Find the resistance of the pencil lead by finding the ratio V/I. **(1 mark)**.

$$\rho = \frac{RA}{L}$$

Use this equation to calculate the resistivity of the pencil lead **(1 mark)**

[8MAX] (8)

(c) During an experiment such as that described in part (b), a specimen of pencil "lead" is found to have a resistance of 52 Ω when the current through it is 250 mA.

Calculate the power dissipated in the specimen under these conditions.

$E = IVt,\ P = IV,\ P = I^2R$ = 0.25^2 x 52 **(1 mark)** = 3.3 W **(1 mark)**

(2)

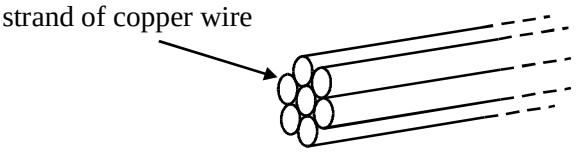
(Total 12 marks)

Q2. (a) Show that the unit of resistivity is Ω m.

$\rho = \frac{RA}{L}$ unit of resistance x unit of area/unit of length
= $\Omega m^2/m$ = Ωm **(1 mark)**

(1)

(b) A cable consists of seven straight strands of copper wire each of diameter 1.35 mm as shown in the diagram.



Calculate

(i) the cross-sectional area of **one strand** of copper wire,

Always change the measurements into metres before you start

d = 1.35 mm = 1.35 x 10^{-3} m

$A = \pi r^2 = \pi (d/2)^2 = \pi (1.35 \times 10^{-3}/2)^2 = 1.43 \times 10^{-6}$ m^2 **(1 mark)**

(ii) the resistance of a 100 m length of the **cable**, given that the resistivity of copper is 1.6×10^{-8} Ωm.

Here you have to realize that you are asked to find the resistance of seven resistors in parallel… the cross section is therefore 7x the answer of part (i)

R = ρL/A = **(1 mark)** 1.6×10^{-8} x 100 /(7 **(1 mark)** x 1.43 x 10^{-6}) **(1 mark)** = 0.16 Ω **QED**

(4)

(c) (i) If the cable in part (b) carries a current of 20 A, what is the potential difference between the ends of the cable?

V=IR = 20 x 0.16= 3.2V **(1 mark)**

(ii) If a single strand of the copper wire in part (b) carried a current of 20 A, what would be the potential difference between its ends?

Seven strands have a resistance that is $1/7^{th}$ of the resistance of one strand. Therefore the resistance of one strand is (7 x 0.16)Ω.

V = IR = 20 x 7 x 0.16 = 22V **(1 mark)**

(2)

(d) State **one** advantage of using a stranded rather than a solid core cable with copper of the same total cross-sectional area.

Any one of:

- A stranded cable is more flexible than one with a solid core.
- If one strand fails, cable continues to conduct electricity whereas if a solid core snapped no current would flow.
- The larger surface area of the stranded cable leads to better heat dissipation.

(1)

(Total 8 marks)

Q3. A particular heating element consists of a 3.0 m length of a metal alloy wire of diameter 1.2 mm (so, radius = 0.6 mm) and resistivity 9.3×10^{-6} Ωm at the element's operating temperature. The element is designed for use with a 230 V supply. Calculate the rating, in W, of the heating element when in use.

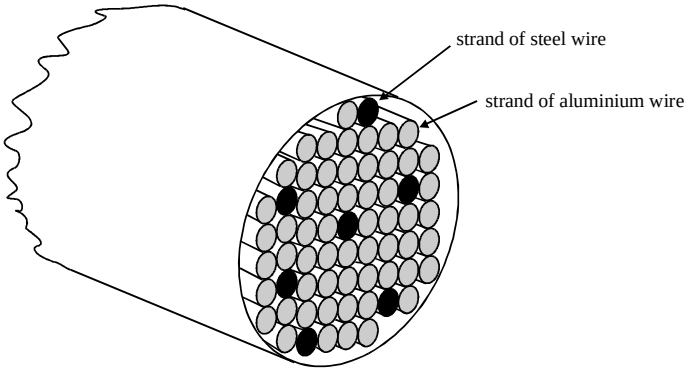
$A = \pi r^2 = \pi \times (0.6 \times 10^{-3})^2 = 1.13 \times 10^{-6}$ (m^2) **(1 mark)**

$R = \frac{\rho l}{A}$ = 24.7Ω **(1 mark)**

$P = \frac{V^2}{R}$ [or I = 9.32 A] **(1 mark)** = 2.1(4) × 10^3 W **(1 mark)**

(Total 4 marks)

Q4. The cable shown in the diagram is used to transmit electricity and is made from strands of steel wire and strands of aluminium wire. The strands of wire are in electrical contact with each other along the length of the cable.



Resistivity of aluminium = 2.65×10^{-8} Ωm

(a) Calculate the resistance of one strand of aluminium wire with a diameter of 3.2 mm (1.6 mm radius) and a length of 1.0 km.

$R \left(= \frac{\rho l}{A} \right) = \frac{2.65 \times 10^{-8} \times 1.0 \times 10^{3} \times 4}{\pi \times (3.2 \times 10^{-3})^2}$ **(1 mark)**

= 3.30 Ω **(1 mark)**

(b) The resistance of one strand of steel wire in a 1.0 km length of cable is 19.9 Ω. Calculate the resistance of 1.0 km of the cable made up of seven strands of steel wire and fifty four strands of aluminium wire.

Resistance of n resistors in parallel = R/n

Resistance of steel strands = 19.9/7 = 2.84Ω **(1 mark)**

Resistance of the aluminium strands = 3.30/54 = 0.0611Ω **(1 mark)**

Resistance of the steel in parallel with the aluminium:

$1/R_{Total} = 1/R_{Steel} + 1/R_{Aluminium}$ = (1/2.84 + 1/0.0611) = 16.72

R_{Total} = 0.060Ω **(1 mark)**

(Total 5 marks)

5. (a) Write down an equation relating the electrical resistivity of a material to the resistance of a particular sample. Define the symbols used.

$$\rho = \frac{RA}{L}$$

ρ = Resistivity of the material R = resistance of specimen, *l* = length, *A* = cross-sectional area **(2 marks if all correct – lose one for each mistake)**

(2)

(b) (i) A cylindrical sample of graphite 10mm long and with diameter 6.0mm (3.0mm radius) has a resistance of 450 Ω. Calculate the electrical resistivity of graphite.

$\rho = 450 \times \pi \times (3.0 \times 10^{-3})^2/(10 \times 10^{-3})$ **(1 mark)** = 1.3 **(1 mark)** Ω m **(1 mark)**

(ii) Calculate the resistance of a specimen of copper ($\rho = 1.7 \times 10^{-8}$ Ωm) of the same dimensions as that of the graphite specimen in part (b)(i).

Put all of the constant values on one side of the equation and the changing ones on the other. ρ_1/R_1 = constant

$$\rho_1/R_1 = \rho_2/R_2$$

$$1.3/450 = 1.7 \times 10^{-8}/R$$

$$R = 450 \times 1.7 \times 10^{-8}/1.3 = 5.9 \times 10^{-6}\ \Omega$$ **(1 mark)**

(5)

(Total 7 marks)

6. A 15 A fuse uses a 55 mm length of copper wire of diameter 0.508 mm (radius = 0.254 mm). Calculate the resistance of the copper wire at 20 °C.

$$\rho = \frac{RA}{L}$$

resistivity of copper at 20 °C = 1.7×10^{-8} Ωm

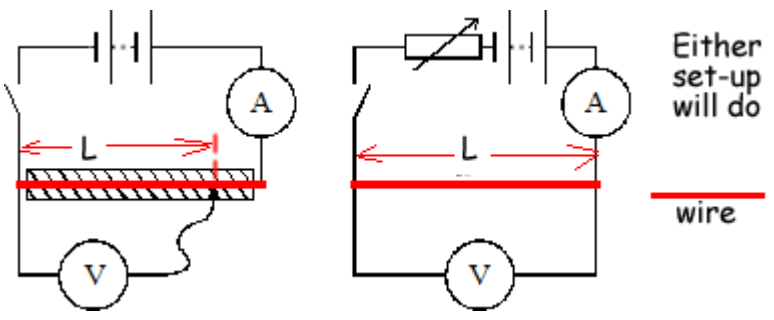
$R = \rho L/A = 1.7 \times 10^{-8} \times 55 \times 10^{-3}/ (\pi \times (0.254 \times 10^{-3})^2)$ **(1 mark)**

$= 4.6 \times 10^{-3}$ **(1 mark)** Ω **(1 mark)**

(Total 3 marks)

7. (a) The resistivity of a material in the form of a uniform resistance wire is to be measured. The area of cross-section of the wire is known. The apparatus available includes a battery (NOT a single cell!), a switch, a variable resistor, an ammeter and a voltmeter.

(i) Draw a circuit diagram using some or all of this apparatus, which would enable you to determine the resistivity of the material.



(1 mark) battery, wire (you could do this as a resistor symbol with the word 'wire' across the top of it), (variable resistor – if you draw the second arrangement) and ammeter in series
(1 mark) voltmeter connected across wire

(ii) Describe how you would make the necessary measurements, (need to say what instrument you use to do this) ensuring that you have a range of values (say you would have at least six to get a trend) and how you would alter the values – move contact – move slider on the variable resistor).

First Circuit

(with switch closed) measure current with the ammeter and potential difference with the voltmeter **(1 mark)**
move contact along the wire **(1 mark)** (or length of wire changed)
measure new (current and) potential difference **(1 mark)**
measure L each time with a metre ruler **(1 mark)** and note down the results.

Repeat this for a range of readings – minimum of six readings **(1 mark)**

Second Circuit

measure current from the ammeter and potential difference from the voltmeter **(1 mark)**
change variable resistor value by moving the slider **(1 mark)**
measure new current and potential difference **(1 mark)**
Measure L with a metre ruler – then it stays the same **(1 mark)**

Repeat this for a range of readings – minimum of six readings **(1 mark)**

(iii) Show how a value of the resistivity is determined from your measurements.

$$\rho = \frac{RA}{L}$$

Circuit 1 –

Calculate resistance (V/I).

Plot a graph of resistance against length and obtain gradient of the graph. **(1 mark)**

Resistivity = gradient x cross-sectional area. **(1 mark)**

OR

Circuit 2 –
Plot a graph of potential difference against current and obtain the gradient of graph - the resistance **(1 mark)**

A and *L are* known, so ρ = gradient of graph x (A/L) **(1 mark)**

(9)

(b) A sheet of carbon-reinforced plastic measuring 80 mm × 80 mm × 1.5 mm has its two large surfaces coated with highly conducting metal film. When a potential difference of 240V is applied between the metal films, there is a current of 2.0 mA in the plastic. Calculate the resistivity of the plastic.

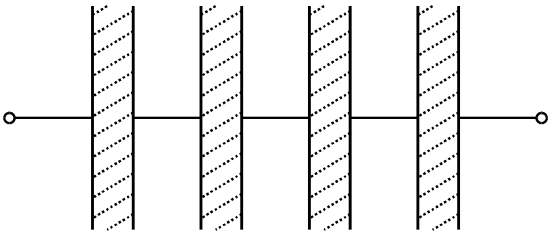
Area = $80 \times 10^{-3} \times 80 \times 10^{-3} = 6.4 \times 10^{-3}$ m^2 and $L = 1.5 \times 10^{-3}$m **(1 mark)**

$R = V/I = 240/(2.0 \times 10^{-3}) = 1.2 \times 10^5 \Omega$ **(1 mark)**

$\rho = RA/L = 1.2 \times 10^5 \times 6.4 \times 10^{-3} / (1.5 \times 10^{-3}) = 5.1 \times 10^5 \Omega m$ **(1 mark)**

(3)

(c) If four of the units described in part (b) are connected as shown in the diagram, calculate the total resistance of the combination.



They are connected in series: **(1 mark)**

$4 \times 1.2 \times 10^5 \Omega = 4.8 \times 10^5 \Omega$ **(1 mark)**

(2)
(Total 14 marks)

8. (a) For a conductor in the form of a wire of uniform cross-sectional area, give an equation which relates its resistance to the resistivity of the material of the conductor.
Define the symbols used in the equation.

$$\rho = \frac{RA}{L}$$

ρ = *Resistivity of the material* R = resistance of specimen, *l* = length, *A* = cross-sectional area **(2 marks if all correct – lose one for each mistake)**

(2)

(b) (i) An electrical heating element, made from uniform nichrome wire, is required to dissipate 500 W when connected to the 230 V mains supply.
The cross-sectional area of the wire is 8.0×10^{-8} m^2. Calculate the length of nichrome wire required. (resistivity of nichrome = 1.1×10^{-6} Ω m)

$P = IV$ but $V=IR$ so $P = V^2/R$ **(1 mark)**
$R = 230^2/500 = = 106\Omega = 110\ \Omega$ - 2sf **(1 mark)**

$L = RA/\rho = (106 \times 8.0 \times 10^{-8})/(1.1 \times 10^{-6}) = 7.7$ m **(1 mark)**

When you work out an intermediate value – do so to 1sf more than the final answer will be quoted to. Use that value to substitute into equations.

(ii) Two heating elements, each rated at 230 V, 500 W are connected to the 230 mains supply

(A) in series, (B) in parallel.

Explain (No calculation is wanted here – they want you to reason it) why only one of the circuits will provide an output of 1 kW.

When they are connected **in parallel** each of them receives a potential difference of 230V across it **(1 mark)** – therefore each of them gives an energy transfer of 500 J/s into heat – a total of 1000 W – 1 kW. **(1mark)**

When connected **in series** they each receive a potential difference of only 115V (they share the voltage) therefore the current through each of them is lower than that they were designed for **(1mark)** and the output will be less than 1kW. **(1mark)** **(MAX 4)**

(6)
(Total 8 marks)

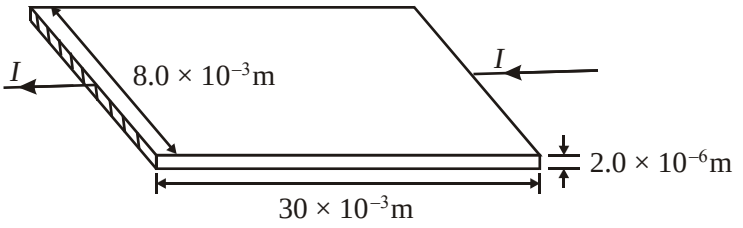
9. (a) Give an expression for the *resistivity* of a material in the form of a uniform wire. Define all the symbols used.

$$\rho = \frac{RA}{L}$$

ρ = *Resistivity of the material* R = resistance of specimen, l = length, A = cross-sectional area **(2 marks if all correct – lose one for each mistake)**

(2)

(b) A thin film of carbon may be used in some electronic systems. Typical dimensions of such a film are shown on the right.

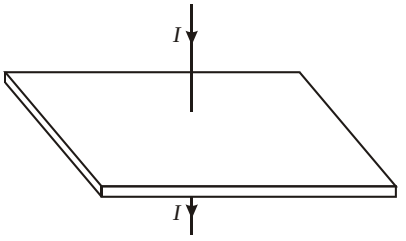


(i) Calculate the resistance of the carbon film to a current I passing through it as shown above.
(resistivity of carbon = $4.0 \times 10^{-5}\ \Omega$ m)

$A = 8.0 \times 10^{-3} \times 2.0 \times 10^{-6} = 1.6 \times 10^{-8}\ m^2$ **(1 mark)** $L = 30 \times 10^{-3}$ m

$R = \rho L/A = (4.0 \times 10^{-5}) \times (30 \times 10^{-3})/(1.6 \times 10^{-8}) = 75\ \Omega$ **(1 mark)**

(ii) Without recalculating the resistance of the carbon film, explain how you would expect the resistance to change if the current flowed as in the diagram on the right. You should consider the numerical ratio or factor by which each dimension affecting the resistance has changed.



L would decrease by a factor of 15,000 (went from 30×10^{-3} to 2.0×10^{-6}) and A would increase by a factor of 15,000. Resistance depends on L/A so it would therefore decrease by a factor of 15000 x 15000 = 225,000,000 **(1 mark)** meaning that current would increase by that factor **(1 mark)**

They do not want you to do this – but here is the working to show that this is the case……

$A = 8.0 \times 10^{-3} \times 30 \times 10^{-3} = 2.4 \times 10^{-4}\ m^2$ $L = 2.0 \times 10^{-6}\ m$

$R = \rho L/A = (4.0 \times 10^{-5}) \times (2.0 \times 10^{-6})/(2.4 \times 10^{-4}) = 3.3 \times 10^{-7}\ \Omega$

(4)

(Total 6 marks)

10. (a) A metal wire of length 1.4 m has a uniform cross-sectional area = $7.8 \times 10^{-7}\ m^2$. Calculate the resistance, *R*, of the wire.

resistivity of the metal = $1.7 \times 10^{-8}\ \Omega m$

$$\rho = \frac{RA}{L}$$

$R = \rho L/A = (1.7 \times 10^{-8}) \times 1.4/(7.8 \times 10^{-7}) = 0.031\ \Omega$

(2)

(b) The wire is now stretched to twice its original length by a process that keeps its volume constant. If the resistivity of the metal of the wire remains constant, show that the resistance increases to 4*R*.

Constant volume means that $L_1A_1 = L_2A_2$

So $L_2 = 2L_1$ and $A_2 = A_1/2$ **(1 mark)**

R_1 = constant x L/A

R_2 = constant x $2L/\frac{1}{2}A$ = constant x $4L/A$ **(1 mark)**

[or calculation with $l_2 = 2.8$ (m) and $A_2 = 3.9$ (m^2) **(1 mark)**]

gives $R = 0.124\ \Omega$ **(1 mark)**]

(2)

(Total 4 marks)